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Construction of a modified butterfly subdivision scheme with C^2-smoothness and fourth-order accuracy

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Abstract

This article presents a modified butterfly subdivision scheme with improved smoothness over regular triangular meshes. The proposed technique is an approximating scheme with a tension parameter. It achieves fourth-order accuracy and generates C^2 limit surfaces for a suitable range of the parameter while maintaining the same support of the original butterfly scheme. To validate the theoretical results, some numerical examples are provided.

Introduction

Subdivision schemes are powerful tools for generating smooth curves and surfaces in CAGD and computer graphics. Starting with a set of control points at level 0, a subdivision scheme generates new sets of control points at level k, $k \in \mathbb{N}$, by applying a set of refinement rules. We call a subdivision scheme *interpolatory* if all the control points at the current level are included in the set of control points at the next level; otherwise, it is termed as an *approximating* scheme. Since the first seminal work (Chaikin's corner-cutting algorithm) in [1], there have been intensive studies on subdivision techniques in the literature over the last few decades; for example, see [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15] and the references therein.

This paper is concerned with bivariate subdivision schemes for designing surfaces. The most well-known interpolatory subdivision method for generating surfaces is the *butterfly* scheme [16] which is characterized by approximation order 4 and C^1 smoothness over a regular triangular mesh. Some variants of the butterfly scheme have been proposed to enhance the quality of surfaces. Dyn et al. [17] suggested a modified version of the butterfly scheme that utilizes a slightly larger neighborhood of control points with a new parameter. In [18], Novara et al. extended the modified butterfly scheme by adding six more points, resulting in C^2 smoothness and the approximation order 6. Moreover, Zorin et al. [19] generalized the modified butterfly scheme to the irregular triangulation. On the other hand, non-stationary [18], [20] butterfly schemes reproducing exponential polynomials were proposed. Very recently, Conti and López-Ureña [21] introduced an essentially non-oscillatory interpolatory subdivision scheme on regular triangular grids, a non-linear analogue of the original butterfly scheme.

Despite these various extensions of the butterfly subdivision scheme, there is no direct and natural extension to have C^2 smoothness, while preserving the same support of the classical butterfly scheme. These schemes include more vertices in the refinement rules to obtain higher continuity, causing some loss of locality in the final limit surfaces. In this regards, we seek a different way to construct a modified butterfly subdivision scheme with improved smoothness on a regular triangular grid. The main idea of our approach is to design our technique as an approximating scheme with a tension parameter (say, ω). Although we sacrifice the interpolating property, for a small parameter value of ω, our technique becomes a nearly interpolating scheme. As a result, it provides fourth-order accuracy, and its basic limit functions have C^2 continuity for a suitable range of ω, while maintaining the same support as the classical butterfly scheme. To the best of our knowledge, approximating butterfly subdivision schemes (for triangular grids) fulfilling such properties have been never proposed before. Although this study does not consider the construction of extraordinary rules, it is still an active field of research. In fact, some applications of 3D biomedical images do not need to handle meshes with extraordinary points [18].

We structure this paper as follows. Section 2 recalls preliminary notation and definitions. In Section 3, we introduce a new butterfly subdivision scheme with a tension parameter. Its convergence and smoothness are analyzed in Section 4. Finally, the approximation order of the proposed scheme is discussed in Section 5.

Section snippets

Notation and definitions

For the presentation of this paper, we use the following notation. For each positive integer n , Π_n indicates the space of all bivariate algebraic polynomials of degree $\leq n$. Denote by $\mathbb{Z}_+$ the set of nonnegative integers and put $\mathbb{Z}_+^2 := \{\alpha := (\alpha_1, \alpha_2) \in \mathbb{Z}^2 : \alpha_1, \alpha_2 \geq 0\}$. Corresponding to $\alpha := (\alpha_1, \alpha_2) \in \mathbb{Z}_+^2$ and $z := (z_1, z_2) \in \mathbb{C}^2$, we set $|\alpha|_1 := \alpha_1 + \alpha_2, \quad \alpha! := \alpha_1!\alpha_2!, \quad z^\alpha := z_1^{\alpha_1} z_2^{\alpha_2}$.Let $\ell^\infty(\mathbb{Z}^2)$ be the space of all bounded sequences defined on $\mathbb{Z}^2$. For a given matrix $\mathbf{A} := (A_{i,j} : i, j = 1, 2) \in \mathbb{R}^{2\times 2}$, denote $|\mathbf{A}| := (|A_{i,j}| : i, j = 1, 2), \quad \|\mathbf{A}\|_\infty := \max\{|A_{i,1}| + |A_{i,2}| : i$...

Improved butterfly subdivision scheme

In this section, we present a novel modified butterfly approximating subdivision scheme (hereafter, termed as 'MBASS'). The refinement rules of the MBASS are based on the configuration of weights illustrated in Fig. 1. Unlike the existing butterfly schemes, the proposed technique is an approximating scheme such that the existing vertices are adjusted at each subdivision step. Specifically, the vertex rule of the MBASS is defined by using the 7-point stencil in Fig. 1 (left): $\mathbf{f}_v^{k+1} = \left(1 - \frac{3\omega}{8}\right) \mathbf{f}_{n_0}^k + \sum_i$...

Smoothness analysis

The purpose of this section is to show that the MBASS generates C^2 limit functions for a suitable range of the parameter ω. Our approach is based on checking the contractivity of the difference schemes derived from the given subdivision operator (see [12], [24]). To this end, for a given operator $\mathsf{S}_\mathbf{B}$ with matrix mask $\mathbf{B}$, we define the norm of $\mathsf{S}_\mathbf{B}^\ell$, $\ell \in \mathbb{N}$, corresponding to ℓ-times iterated steps as follows: $\|\mathsf{S}_\mathbf{B}^\ell\|_\infty := \max\left\{\left\|\sum_{\beta\in\mathbb{Z}^2} \left|\mathbf{B}^{[\ell]}{}_{\alpha-2^\ell\beta}\right|\right\|_\infty : \alpha \in \left[0, 2^\ell - 1\right]^2 \cap \mathbb{Z}_+^2\right\}$.Indeed, the norm of an ℓ-times iterated...

Approximation order

This section is concerned with the approximation order of the MBASS. When a convergent subdivision scheme reproduces polynomials in Π_d, it can achieves approximation order $d+1$. However, the order of accuracy derived from the polynomial reproduction degree is, in general, not optimal (e.g., see [23], [25]). For instance, the B-spline subdivision of degree $d \in \mathbb{N}$ reproduces polynomials in Π_1 only, but it can retain the approximation order $d+1$ by applying a suitable operator to the given sequence [25]…

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